

Temporal Dark Solitons in Nonuniform Bose-Einstein Condensates

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Abstract

We discuss temporal dark solitons in confined nonuniform Bose condensates. As a kind of localized high excitations, these solitons can be viewed as macroscopic quasiparticles, having relative motion to the background condensate. We get analytic expression for one dark soliton under slowly varying approximation and discuss its special propagation properties in nonuniform condensate, then we numerically prove that this approximation is reasonable and this kind of solitons exhibit their propagation properties in the nonuniform condensate. Finally, we simulate the generation of dark-soliton-like pulses in the condensate, and indicate that the excitation experiment, done by W. Ketterle's group [14], can also be interpreted in terms of temporal dark soliton.

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Bose Einstein condensation(BEC) has been observed in dilute alkali atomic vapors [1, 2, 3]. Recently, further experiments have also demonstrated that the condensate can be perfectly described by the Gross-Pitaevskii equation [4, 5, 6, 7]. For a ground-state condensate, the kinetic energy in this equation is usually negligible as compared with the s-wave scattering interaction energy that is the cubic nonlinear term. This indicates that the scattering term dominates main properties of the ground-state condensate, although it is a weakly interacting system. But for an excited condensate in which macroscopic number of atoms are collectively excited from ground state into high modes, the kinetic energy may become comparable to the scattering interaction energy. It could be noticed that the Gross-Pitaevskii equation, except for the confining potential, takes the form of the cubic nonlinear Schrödinger equation (CNLSE) which is well-known for the existence of soliton solutions [8]. Naturally, one will ask whether solitons can exist in a condensate or not. In the previous works, A. Mysyrowicz et al. have already found the soliton-like propagation of the condensed excitons in Cu_2O crystal [9], which obeys a similar Gross-Pitaevskii equation predicted by E. Hanamura [10]. Weiping Zhang et al., G. Lenz et al., and P. A. Ruprecht et al. have proved the existence of bright soliton in coherent atomic waves [11, 12] and condensate [7] respectively. S. A. Morgan et al. have analyzed the solitary wave, synchronously moving with the background condensate [13]. The purpose of this paper is to discuss temporal dark solitons, as a kind of macroscopic quasiparticles, which have relative motion to the background condensate, and it is different from the work in reference [13]. This kind of solitons is actually localized high-mode excitations in the condensate consisting of atoms with positive scattering length, such as ^{87}Rb or ^{23}Na . First, we will consider a single temporal dark soliton in an idealized uniform condensate. Second, we will consider it in a nonuniform condensate and discuss its specific propagation features in this case. Third, we will numerically simulate the propagation and collision of this kind of solitons in nonuniform condensate in order to prove that the dark solitons can exist in the nonuniform condensate steadily, exhibiting those propagation features, and the approximation adopted in the analytical solution is reasonable. Finally, we will also simulate the generation of dark-soliton-like pulses by arbitrary perturbations, using the dark solitons' nonlinear superposition properties, and indicate that the excitation experiment, done by W. Ketterle's group [14], can also be interpreted in terms of temporal dark soliton.

For a highly excited condensate in which macroscopic number of atoms are excited into

high excited modes, the creation and destruction operators of these modes can be considered to be able to commute with each other [15]. Therefore, in an idealized uniform condensate, its atomic field operator can be treated as scalar, called macroscopic wave function. The macroscopic wave function can be described by the following Gross-Pitaevskii equation [16],

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V_{\text{exc}} \Psi(\mathbf{r}, t) + U_0 |\Psi(\mathbf{r}, t)|^2 \Psi(\mathbf{r}, t) \quad (1)$$

where $\Psi(\mathbf{r}, t)$ is the macroscopic wave function of the condensate, including not only the ground state but also the macroscopically populated high excited modes. V_{exc} is an external flat potential. $U_0 = 4\pi\hbar^2 a_{sc}/m$ is a scattering constant, where a_{sc} is the s-wave scattering length and m is the atomic mass. Eq. (1) is a CNLSE that possesses soliton solutions. When $U_0 > 0$ for positive scattering length, the solutions should be nonlinear superposition of dark solitons, according to the soliton theorems [8]. With the inverse scattering transform method [8], we can solve this equation and get one-soliton solution,

$$\Psi(\mathbf{r}, t) = \Psi_0 \frac{1 + (\mu_s - i\nu_s)^2 e^{-2\Gamma}}{1 + e^{-2\Gamma}} e^{-i(V_{\text{exc}} + U_0 |\Psi_0|^2)t/\hbar} \quad (2)$$

where Ψ_0 is the background amplitude of $\Psi(\mathbf{r}, t)$, μ_s and ν_s are real constants, satisfying $\mu_s^2 + \nu_s^2 = 1$, and μ_s is called the eigenvalue of this dark soliton. In this equation,

$$\Gamma = \nu_s \left[\sqrt{U_0 |\Psi_0|^2 m/\hbar^2} \mathbf{k}_s \cdot (\mathbf{r} - \mathbf{r}_0) + \mu_s U_0 |\Psi_0|^2 t/\hbar \right] \quad (3)$$

where \mathbf{k}_s is a unit vector in the propagation direction of the soliton, \mathbf{r}_0 is the center coordinate of the soliton. Therefore, the atomic number density distribution is

$$|\Psi(\mathbf{r}, t)|^2 = |\Psi_0|^2 (1 - \nu_s^2 \text{sech}^2 \Gamma) \quad (4)$$

where ν_s^2 is called the darkness of the dark soliton. Additionally, it is worthy to be emphasized that dark soliton possesses no threshold, so it can be stimulated easily. According to the theorems of dark soliton [17], an arbitrary perturbation of the wave function can be described as a nonlinear superposition of solitons, which indicates that soliton has its universality in the space described by the CNLSE (1). From expression (4), we can find that the soliton is a localized function, because when $z \rightarrow \pm\infty$, its dark density decreases to zero quickly. Thus we can derive its full width at half peak darkness,

$$\Delta_{\text{s(FWHM)}} = \frac{\text{arccosh}\sqrt{2}}{\nu_s \sqrt{\pi a_{sc} |\Psi_0|^2}} \quad (5)$$

This expression has a similar form with the healing length of the Bose condensate [18], except for the characteristic constant of the soliton. It is known that a two-dimensional vortex core in the Bose condensate has a similar size to the healing length. Consequently, their common features tell us that the dark soliton could be viewed as a kind of one-dimensional vortex core. Although there is no circular current around this kind of core due to its noncircularly-connected feature, the both sides of the soliton are connected by its bottom current density, which we will give more discussion later. Similar to a two-dimensional vortex, this one-dimensional vortex has a “vortex line”, or more exactly, it may be called vortex plane, which is the center plane of the soliton perpendicular to the vector \mathbf{k}_s . Corresponding to the circulation around a vortex line in two-dimensional case, there is also a similar quantity for the one-dimensional vortex core, which is just the multiply of the width and the velocity of the soliton and is directly determined by its characteristic constant. However, for a dark soliton, its characteristic constant is usually not a quantized, but a continuously valued number, this feature indicates that the one-dimensional vortex is not quantized for an uniform Bose condensate, and therefore the corresponding one-dimensional flow should exhibit classical fluidity instead of superfluidity. But, this does not mean that it won't become quantized for the externally confined nonuniform Bose condensate, because in considering the solitonic nonlinear dynamics at the vicinity of the confined boundaries, this problem always becomes very complicated and couldn't be solved analytically at present. With Eq. (5), we can calculate that when the atomic number density of the condensate $|\Psi_0|^2$ is $10^{14}cm^{-3}$, the width of a black ($\nu_s = 1$) soliton is $0.95\mu m$ for condensate of ^{23}Na . Obviously, it can be much smaller than the real size of a nonuniform condensate, for example, $17\mu m$ in radial direction and $300\mu m$ in axial direction, realized by W. Ketterle's group [2]. So the previous idealized uniform condensate is reasonable for very dark solitons.

Subsequently, we consider a more real and special case in which the condensate, confined in external potential, is nonuniform, and it has a shape of cigar. Similar to the above assumption, a macroscopic number of atoms are still assumed to be excited, and the commutation relations of the atomic field operators are still valid. Thus, the Gross-Pitaevskii equation can be expressed as

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V_{\text{ext}}(\mathbf{r}) \Psi(\mathbf{r}, t) + U_0 |\Psi(\mathbf{r}, t)|^2 \Psi(\mathbf{r}, t) \quad (6)$$

where $V_{\text{ext}}(\mathbf{r}) = V_r(\mathbf{R}) + V_a(z)$ is the external confining potential, which can be divided into

two parts, the radial $V_r(\mathbf{R})$ and the axial $V_a(z)$ in cylindrical coordinates. Additionally, We assume the wave function $\Psi(\mathbf{r}, t)$ can be divided into the background ground-state condensate $\Phi(\mathbf{r})$ and a soliton function $\beta(z, t)$ propagating along z axis,

$$\Psi(\mathbf{r}, t) = \beta(z, t) \Phi(\mathbf{r}) e^{-i\mu_c t/\hbar} \quad (7)$$

where μ_c is the chemical potential of the stationary ground-state of the condensate. Then Eq. (6) is transformed into

$$\begin{aligned} & \left[i\hbar \frac{\partial \beta(z, t)}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \beta(z, t)}{\partial z^2} + \frac{\hbar^2}{m} \frac{\partial \beta(z, t)}{\partial z} \frac{\partial \ln \Phi(\mathbf{r})}{\partial z} \right] \Phi(\mathbf{r}) \\ & + [-U_0 |\beta(z, t) \Phi(\mathbf{r})|^2 \beta(z, t) + U_0 |\Phi(\mathbf{r})|^2 \beta(z, t)] \Phi(\mathbf{r}) \\ = & \beta(z, t) \left[-\mu_c \Phi(\mathbf{r}) - \frac{\hbar^2}{2m} \nabla^2 \Phi(\mathbf{r}) + V_{ext}(\mathbf{r}) \Phi(\mathbf{r}) + U_0 |\Phi(\mathbf{r})|^2 \Phi(\mathbf{r}) \right] \end{aligned} \quad (8)$$

Because $\Phi(\mathbf{r})$ is the stationary ground-state wave function of the condensate, it must satisfy

$$\mu_c \Phi(\mathbf{r}) = -\frac{\hbar^2}{2m} \nabla^2 \Phi(\mathbf{r}) + V_{ext}(\mathbf{r}) \Phi(\mathbf{r}) + U_0 |\Phi(\mathbf{r})|^2 \Phi(\mathbf{r}) \quad (9)$$

Consequently, Eq. (8) is reduced to

$$\begin{aligned} & i\hbar \frac{\partial \beta(z, t)}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \beta(z, t)}{\partial z^2} + \frac{\hbar^2}{m} \frac{\partial \beta(z, t)}{\partial z} \frac{\partial \ln \Phi(\mathbf{r})}{\partial z} \\ & - U_0 |\beta(z, t) \Phi(\mathbf{r})|^2 \beta(z, t) + U_0 |\Phi(\mathbf{r})|^2 \beta(z, t) = 0 \end{aligned} \quad (10)$$

Furthermore, we assume $|\Phi(\mathbf{r})|^2$ is a slowly varying function of coordinate z , as compared with $\beta(z, t)$, satisfying

$$\frac{1}{2} \left| \frac{\partial^2 \beta(z, t)}{\partial z^2} \right| \gg \left| \frac{\partial \beta(z, t)}{\partial z} \frac{\partial \ln \Phi(\mathbf{r})}{\partial z} \right| \quad (11)$$

Therefore, we can ignore the third term in Eq. (10), and get

$$i\hbar \frac{\partial \beta(z, t)}{\partial t} + \frac{\hbar^2}{2m} \frac{\partial^2 \beta(z, t)}{\partial z^2} - U_0 |\beta(z, t) \Phi(\mathbf{r})|^2 \beta(z, t) + U_0 |\Phi(\mathbf{r})|^2 \beta(z, t) = 0 \quad (12)$$

Because $|\Phi(\mathbf{r})|^2$ is a slowly varying function in contrast to a soliton, when we solve this equation with inverse scattering transform method for a single soliton solution, we can take $|\Phi(\mathbf{r})|^2$ as a constant. Subsequently, we can routinely get one soliton expression,

$$\beta(z, t) = \frac{1 + (\mu_s - i\nu_s)^2 e^{-2\Gamma}}{1 + e^{-2\Gamma}} \quad (13)$$

where

$$\Gamma = \nu_s \left[\sqrt{U_0 |\Phi(\mathbf{r})|^2 m / \hbar^2} \mathbf{k}_s \cdot (\mathbf{r} - \mathbf{r}_0) + \mu_s U_0 |\Phi(\mathbf{r})|^2 t / \hbar \right] \quad (14)$$

μ_s and ν_s still follow the previous definition. z_0 is the center coordinate of this soliton. The full width at half peak darkness of the soliton is

$$\Delta_{s(\text{FWHM})} = \frac{\text{arccosh}\sqrt{2}}{\nu_s \sqrt{\pi a_{sc} |\Phi(\mathbf{r})|^2}} \quad (15)$$

which is different from Eq. (5) for $|\Phi(\mathbf{r})|^2$ being a slowly varying function of z . This means $\Delta_{s(\text{FWHM})}$ is to be increased when the soliton moves from the “top” to the “downhill” of the ground condensate wave function. From Eq. (14) we can derive the velocity of the soliton,

$$\mathbf{v}(\mathbf{r}) = -\mu_s \sqrt{\frac{U_0 |\Phi(\mathbf{r})|^2}{m}} \mathbf{k} \quad (16)$$

where \mathbf{k} is the unit vector in the direction of z . $\mathbf{v}(\mathbf{r})$ varies with $\Phi(\mathbf{r})$, which is consistent with the local speed of sound given by Bogoliubov [19] and Lee, Huang, and Yang [20]. Additionally, Eq. (16) tells us that the velocity of this temporal dark soliton is also relevant to the eigenvalue, and it is, more explicitly, usually less than the absolute value of the corresponding sound speed in the nonuniform Bose condensate. However, this property can not be gotten in the density perturbation theory of hydrodynamics, such as ref. [21]. In considering the local density dependent property of the velocity, we can find that there is a slow feedback process between the velocity and the displacement of the soliton, and the temporal dark soliton behaves as an oscillator in the nonuniform Bose condensate [22]. Additionally, for those radial tightly confined Bose condensates, the velocity of the soliton varies rapidly with the density in the radial direction. This leads to very serious radial dispersion of the temporal dark soliton, to which a similar phenomenon has been observed in the experiments done by W. Ketterle’s group [14]. As it is known in nonlinear optics, the $1 + 1$ dimensional dark solitons are usually unstable under their transverse perturbations, and always evolve into dark vortices [23]. The analogy between this solitonic excitation in Bose condensate and the optical dark solitons enlighten us that the temporal dark soliton must be unstable under the stretch of transverse dispersion, and it may also evolve into vortices. Due to the cylindrical symmetry of the ground state condensate, it is more likely to evolve into vortex rings [16], because they can still preserve the cylindrical symmetry. The circulation of those stable vortex rings must be quantized, which is a direct consequence of

the coherent property of the Bose condensate, relevant to the nature of the superfluidity of the Bose condensate. Our further work on this topic is still in progress. Assuming that the radial variation could be neglected for some cases, we will find the product

$$\Delta_{s(\text{FWHM})} \cdot v(\mathbf{r}) = -2\text{arccosh}\sqrt{2}\frac{\hbar\mu_s}{m\nu_s} \quad (17)$$

should be a constant for a darkness-fixed soliton. As we have described above, analogous to the circulation around the vortex line of a two-dimensional vortex in the fluid, this quantity may also be called a one-dimensional circulation, which characterizes the properties of the one-dimensional dark soliton vortex core directly. But the more important thing is that it is very realistic for an experimental measurement of this quantity. Therefore, Eq. (17) may offer us a useful method to judge whether a dark pulse in the condensate can be very precisely interpreted as a temporal dark soliton. From Eq. (7) we can also get the atomic current density carried by the soliton

$$\mathbf{j} = -\frac{i\hbar}{2m}(\Psi^*(\mathbf{r}, t)\nabla\Psi(\mathbf{r}, t) - \Psi(\mathbf{r}, t)\nabla\Psi^*(\mathbf{r}, t)) = \mathbf{k}\mu_s\nu_s^2U_0^{1/2}m^{-1/2}\Phi^3(\mathbf{r})\text{sech}^2\Gamma \quad (18)$$

which is pulsed. The width of this pulse is proportional to that of the soliton. It is worthy to be noticed that the direction of the current density pulse is opposite to the propagation direction of the dark soliton, however, the propagation direction and the velocity of the current density pulse are both same to those of the soliton. When $\nu_s = 0$ or $\nu_s = 1$, the amplitude of this current density pulse reaches its minimum $j_{\min} = 0$. When $\nu_s = 2\sqrt{3}/9$, it gets to its maximum

$$j_{\max} = \frac{4\sqrt{69}}{243}U_0^{1/2}m^{-1/2}\Phi^3(\mathbf{r}) \quad (19)$$

The existence of this maximum shows that each distribution of the current density usually corresponds to two possible soliton eigenvalues and therefore two one-dimensional dark soliton vortices. This property is different from the usual two-dimensional vortex case. Additionally, from Eq. (18), we can find that the relationship of the current density and the soliton eigenvalue μ_s is in the shape of S. In considering some nonlinear dynamical process, such as the solitonic excitation or the soliton collision with a boundary. This relationship may provide us an opportunity of finding bistable phenomena of the temporal dark soliton.

In the above derivation, we have adopted slowly varying approximation (11). To make sure this approximation is reasonable and temporal dark solitons can really exhibit their propagation properties in a confined nonuniform condensate, we numerically simulate the

propagation and collision of this kind of solitons. We assume the ground-state condensate is composed of $N = 5 \times 10^6$ of ^{23}Na atoms with $a_{sc} = 27.5\text{\AA}$ [24], and the radial trapping frequency is $\omega_r = 2\pi \times 1800\text{Hz}$, the axial trapping frequency is $\omega_a = 2\pi \times 18\text{Hz}$. During the simulation, we adopt one-dimensional Thomas-Fermi approximation in the radial direction of the cylindrically symmetric condensate. Consequently, we can approximate Eq. (6) as one-dimensional Gross-Pitaevskii equation by integration in the radial cross section of the condensate,

$$i\hbar \frac{\partial \psi_a(z, t)}{\partial t} + \frac{2}{3} \mu_c \psi_a(z, t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_a(z, t)}{\partial z^2} + V_a(z) \psi_a(z, t) + \frac{2}{3} U_0 |\psi_a(z, t)|^2 \psi_a(z, t) \quad (20)$$

where $\psi_a(z, t)$ is the axial wave function of the condensate, distributed along the axial direction z , and $V_a(z) = m\omega_a^2 z^2/2$. Although the one-dimensional Thomas-Fermi approximation is still not strict, it can still keep the axial wave function $\psi_a(z, t)$ without slowly varying approximation (11) in the direction of z , and therefore Eq. (20) is sufficient to give the approximation a test. We numerically solve Eq. (20) by split-Fourier transform method [25]. As an example, two solitons' collision process is illustrated in Fig.1. The two soliton, $\nu_{s1} = 0.5$, $\nu_{s2} = 0.5$, set initially on the “downhill” of the condensate wave function with distance $6.2\mu\text{m}$, shown in Fig.1.(a). It should be noted that both the wave function and the atomic number density, shown in all figures in this paper, have already been transformed into dimensionless forms with respect to their ground-state center values at $z = 0.0\mu\text{m}$. These two solitons propagate and collide with each other, and then they separate and propagate independently, as shown in Fig.1.(b). Before and after the collision, they can conserve their shape steadily. From the contour graph of this collision, as shown in Fig.1.(c), the widths of the solitons vary with their position, which indicates the locality of their widths. The velocities of the two solitons are all about $v = 15\mu\text{m}/\text{ms}$. There is no damage occurred to both of these solitons, so the collision must be elastic. More numerical experiments show that so long as the numerical precision is enough, the solitons can propagate steadily as far as one can compute. Therefore, we can draw a conclusion that the slowly varying approximation (11) is reasonable, and the solitons indeed have the propagation properties, indicated above in nonuniform condensate confined by external potential. Additionally, as we have seen, the properties of these temporal dark solitons in the condensate are very similar to those of conventional particles, so we can consider them as a kind of quasiparticles. But they are different from phonons, because every one of them is macroscopic whereas a phonon is

usually microscopic, and furthermore every soliton must be a coherent structure as described in Eq. (13).

According to the dark soliton theorems [8], dark solitons have nonlinear superposition properties. We may use these properties to generate temporal dark solitons in nonuniform condensate. To illustrate this possibility, we numerically simulate the generation of soliton-like pulses by arbitrary perturbations in the nonuniform condensate. As an example, here we give two reversed Gaussian function perturbations in the condensate, shown in Fig.2.(a). The darkness of these two perturbations are 0.44 and 0.64 respectively, and the full widths of them are $2.47\mu m$ and $0.99\mu m$ respectively, and their distance is $10\mu m$. As shown in Fig.2.(b), each of these two dark perturbations splits quickly into more pairs of dark pulses, propagating with different velocity depending on its own darkness. The collision of the two out-split dark pulses must be elastic because the two pulses can still keep even symmetry with their own twins. Obviously, these properties are very similar to those of solitons. From Fig.2.(c), which is the contour graph of Fig.2.(a), we can find that all the velocities of these out-split soliton-like pulses are around $V = 20\mu m/ms$. Although we can't make sure that these out-split pulses won't split anymore (in fact, the pair, splited by the left one, have already begun to split again into more pairs of pulses), with the above elastic collision evidence, we can at least draw a conclusion that the initial dark pulses can be approximated as multi-solitons, each of which is composed of a number of solitons. The recent experiment, done by W. Ketterle's group [14], has shown a similar phenomenon as we have simulated here. In their experiment, one dark perturbation splits into two, and then these two out-split pulses continue to propagate with spreading widths. Although there is no description about observation of further splitting of the out-split dark pulse, the out-split dark pulse must be composed of more than one solitons. The spreading of the pulses can be interpreted as the radial dispersion as we have described above, but one can't exclude some other possible reasons, such as further splitting of the multi-solitons and the widening with the decrement of the background condensate as described by Eq. (15). And it can't be usually simply explained only as the last one, except that one can, by chance, get twin strict dark solitons, every one of which must have the form as we have described analytically in Eq. (13). However, further splitting may not be observable, because the size of the condensate is limited, the velocities differences are all to decrease to zero due to the Eq. (16), and they may therefore have no enough distance to propagate for further splitting.

To test the soliton-like properties of the dark pulses in the present experiment, according to our numerical simulation, we suggest that one can generate two dark pulses propagating in opposite direction, and observe their collision. One can also test these properties by measuring the relationship between the velocity of every steadily propagating out-split dark pulse and its width, and Eq. (17) can be used as a judgment. Certainly, more precise interpretation of the experiment would need more study in details and more realistic simulation which can't be included in this short paper. But, as we have briefly discussed above, the temporal dark soliton theorems are promising tools to interpret the experiment done by W. Ketterle's group [14].

In conclusion, we have derived the temporal dark soliton in nonuniform condensate under slowly varying approximation. Viewed as the one-dimensional vortex cores, these temporal solitons may provide deeper insight into the superfluidity in the cigar-shaped Bose-Einstein condensate. Then we have numerically simulated the propagation and collision of this kind of solitons, and proved that the slowly varying approximation is reasonable and the solitons can exhibit the special propagation properties in the nonuniform condensate. We have finally simulated the generation of soliton-like pulses by two arbitrary dark pulses in the nonuniform condensate, and with this simulation, we have given the experimental results of W. Ketterle's group a qualitative interpretation in terms of temporal dark soliton.

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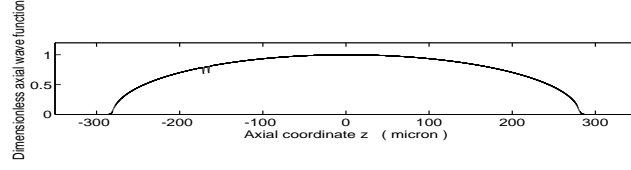


Fig. 1 a "The Propagation of Sound Dark Solitons in Bose-Einstein Condensates"

T. Hong, et al.

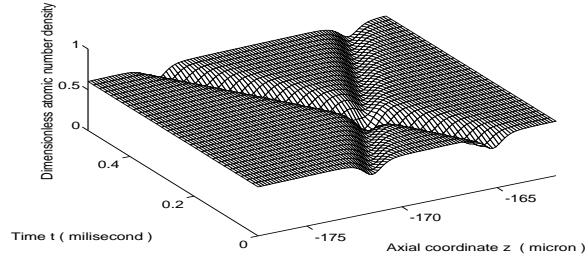


Fig. 1 b "The Propagation of Sound Dark Solitons in Bose-Einstein Condensates"

T. Hong, et al.

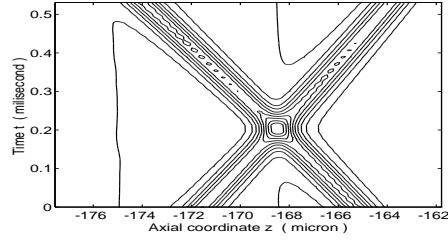


Fig. 1 c "The Propagation of Sound Dark Solitons in Bose-Einstein Condensates"

T. Hong, et al.

FIG. 1: The collision of two temporal dark solitons in the condensate. (a) The dimensionless axial wave function of the ground- state condensate is initially excited with two dark solitons. (b) The two temporal dark solitons propagate steadily and collide with each other elastically. (c) The contour graph of the two colliding solitons. Both the widths of them vary with their position on the condensate.

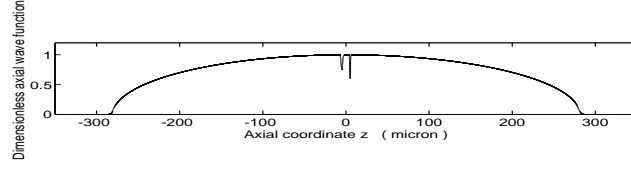


Fig. 2 a "The Propagation of Sound Dark Solitons in Bose-Einstein Condensates"

T. Hong, et al.

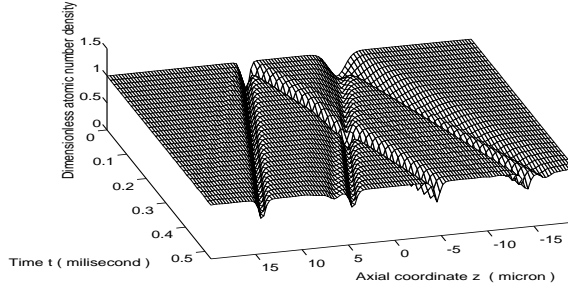


Fig. 2 b "The Propagation of Sound Dark Solitons in Bose-Einstein Condensates"

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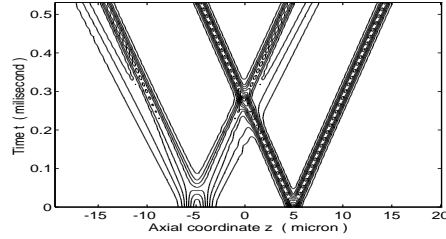


Fig. 2 c "The Propagation of Sound Dark Solitons in Bose-Einstein Condensates"

T. Hong, et al.

FIG. 2: The excitation of dark temporal dark soliton in a ground- state condensate.

(a) The dimensionless axial wave function of the ground- state condensate is initially excited with two reversed Gaussian shape dark pulses.

(b) Each of two dark pulses splits into pairs of dark pulses, and the split pulses collide with each other elastically. This indicates that the initial dark pulses can be approximated as nonlinear superposition of temporal dark solitons.

(c) The contour graph of the two dark pulses. Some of the split dark pulses propagate steadily and some of them are splitting into more dark pulses.